7

CONSERVATION OF ENERGY

EXERCISES

Section 7.1 Conservative and Nonconservative Forces

10. **INTERPRET** In this problem we want to find the work done by the frictional force in moving a block from one point to another over two different paths. Friction is not a conservative force, so mechanical energy is not conserved.

DEVELOP Figure 7.15 is a plan view of the horizontal surface over which the block is moved, showing the paths (a) and (b). The force of friction is $f_k = \mu_k n = \mu_k mg$ (see Equation 5.3) and is directed opposite to the displacement. Because f_k is constant, we use Equation 6.11,

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

Because the friction force is directed opposite to the displacement $d\vec{r}$, the scalar product introduces a negative sign. For path (a), Equation 6.11 takes the form

$$W_a = -\int_{x_1}^{x_2} f_k dx - \int_{y_1}^{y_2} f_k dy = -f_k\left(x_2 - x_1\right) - f_k\left(y_2 - y_1\right)$$

where $x_1 = 0$, $y_1 = 0$, $x_2 = \text{L}$, $y_2 = \text{L}$. For path (b), we use radial coordinates, and Equation 6.11 takes the form

$$W_b = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = -f_k \Delta r$$

where $\Delta r = \sqrt{L^2 + L^2} = \sqrt{2}L$, and the scalar product gives the negative sign because friction always acts opposite to the displacement.

EVALUATE The work done by friction along path (a) is thus

$$W_a = -\mu_k mg(2L)$$

The work done by friction along path (b) is

$$W_b = -\sqrt{2}\mu_k mgL$$

ASSESS Because the work done depends on the path chosen, friction is not a conservative force.

11. **INTERPRET** This problem involves calculating the work done by a conservative force (gravity) and comparing the result obtained for the work done over two different paths.

DEVELOP Take the origin at point 1 in Fig. 7.15 with the x axis horizontal to the right and the y axis vertically upward. Use the same equation for work as we did in Problem 7. 10 (Equation 6.11), but this time the force involved is the force of gravity: $\vec{F}_g = -mg\hat{j}$. For path (a), we use Cartesian coordinates, so $d\vec{r} = dx\hat{i} + dy\hat{j}$. Inserting $\vec{F}_g$ into Equation 6.11 for path (a) thus gives

$$W_a = -\int_{x_1}^{x_2} \overbrace{\left(mg\hat{j}\right) \cdot dx\hat{i}}^{=0} - \int_{y_1}^{y_2} \left(mg\hat{j}\right) \cdot dy\hat{j} = -mg\left(y_2 - y_1\right)$$

For path (b), we will use radial coordinates, so Equation 6.11 takes the form

$$W_b = -\int_{r_1=0}^{r_2=\sqrt{2}L} mg\hat{j}\cdot d\vec{r} = -\int_{r_1=0}^{r_2=\sqrt{2}L} mg\cos\left(45^\circ\right)dr = -\frac{1}{\sqrt{2}}\int_{r_1=0}^{r_2=\sqrt{2}L} mgdr$$

EVALUATE Inserting the initial and final positions into the expression for path (a) gives $W_a = -mgL$. For path (b), we find

$$W_b = -\frac{mg}{\sqrt{2}}\left(r_2 - r_1\right) = -\frac{mg}{\sqrt{2}}\left(\sqrt{2}L - 0\right) = -mgL$$

ASSESS The work done by gravity is the same for both paths, because gravity is a conservative force.

Section 7.2 Potential Energy

12. **INTERPRET** The problem is about gravitational potential energy relative to a reference point of zero energy. In Example 7.1, the reference point was taken to be the 33rd floor. In this problem, we take the street level to be our reference point.

DEVELOP The change in potential energy as a function of vertical distance Δy is given by Equation 7.3, $\Delta U = mg\Delta y$. Each floor is 3.5 m high.

EVALUATE **(a)** The office of the engineer is on the 33rd floor, or is 32 stories above the street level (the first floor) where $U_1 = 0$. Thus, the difference in gravitational potential energy is

$$U_{33} = U_{33} - U_1 = mg\left(32 \text{ floors}\right)\left(3.5 \text{ m/floor}\right) - 0 = \left(55 \text{ kg}\right)\left(9.8 \text{ m/s}^2\right)\left(32\right)\left(3.5 \text{ m}\right) = 60 \text{ kJ}$$

to two significant figures.

(b) At the 59th floor, $U_{59} = mg\left(58 \text{ floors}\right)\left(3.5 \text{ m/floor}\right) = \left(55 \text{ kg}\right)\left(9.8 \text{ m/s}^2\right)\left(58\right)\left(3.5 \text{ m}\right) = 110 \text{ kJ}$, to two significant figures.

(c) Street level is the zero of potential energy, so $U_1 = 0$.

ASSESS Potential energy depends on the reference point chosen, but potential energy difference between two points does not. What matters physically is the difference in potential energy. The differences in potential energy between any two levels are the same as in Example 7.1. For example, $U_{59} - U_{33} = \left(109 \text{ kJ} - 60.4 \text{ kJ}\right) = 49 \text{ kJ}$.

13. **INTERPRET** This problem involves finding the potential energy difference between sea level and locations at different heights above sea level.

DEVELOP The zero of potential energy is at sea level. Use Equation 7.3, $\Delta U = mg\Delta y$, to find the potential energy difference at the other locations.

EVALUATE (a) Atop Mount Washington, the potential energy difference is $\Delta U = (70 \text{ kg})(9.8 \text{ m/s}^2)(1900 \text{ m}) = 1.3$ MJ.

(b) In Death Valley, $\Delta y = -86$ m, so the potential energy difference is $\Delta U = (70 \text{ kg})(9.8 \text{ m/s}^2)(-86 \text{ m}) = -59$ kJ.

ASSESS Notice that the potential energy difference is negative at Death Valley compared to sea level, because Death Valley is below sea level.

14. **INTERPRET** We are asked to find the gravitational potential energy of a person at two different altitudes, using sea level as the zero of potential energy.

DEVELOP From Equation 7.3, we know that gravitational potential energy is $U = mgh$, where h is the height above sea level.

EVALUATE Inserting the given quantities into the expression for potential energy gives

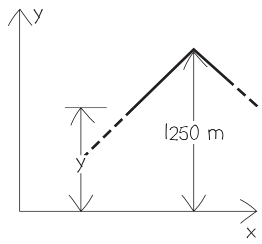
(a) $U = mgh = \left(65 \text{ kg}\right)\left(9.8 \text{ m/s}^2\right)\left(11\times10^3 \text{ m}\right) = 7.0\times10^6 \text{ J} = 7.0 \text{ MJ}$

(b) $U = mgh = \left(65 \text{ kg}\right)\left(9.8 \text{ m/s}^2\right)\left(1.6\times10^3 \text{ m}\right) = 1.0\times10^6 \text{ J} = 1.0 \text{ MJ}$

ASSESS These may seem rather large, but remember that 1 kg, at a height of 1 m, is nearly 10 J. The Joule is not a large unit, and these are large heights.

15. **INTERPRET** The problem is about the change in gravitational potential energy as the hiker ascends. Given the position of zero potential energy, we are interested in her altitude.

DEVELOP The change in potential energy with a change in the vertical distance Δy is given by Equation 7.3, $\Delta U = mg\Delta y = mg(y - y_0)$. Knowing ΔU and y_0 allows us to determine y, see the figure below.



EVALUATE Equation 7.3 gives

$$\Delta U = U(y) - U(y_0) = mg(y - y_0)$$

From the above expression, we find the altitude of the hiker to be

$$y = y_0 + \frac{\Delta U}{mg} = 1250\text{ m} + \frac{-240\text{ kJ}}{(60\text{ kg})(9.8\text{ m/s}^2)} = 840\text{ m}$$

ASSESS In this problem, the point of zero potential energy is taken to be the top of the mountain with $y_0 = 1250$ m. Since the hiker's potential energy is negative, we expect the hiker's altitude to be lower than y_0.

16. **INTERPRET** This problem involves the elastic potential energy, which is a conservative force. We are to find the potential energy we can store in a spring for the given compression and spring constant.
DEVELOP The elastic potential energy of a spring is given by Equation 7.4, $U = kx^2/2$.
EVALUATE Inserting the known quantities into this expression for potential energy gives U = (320 N/m)(0.18 m)^2/2 = 5.2 J.
ASSESS We report the answer to two significant figures because the data is given to two significant figures.

17. **INTERPRET** This problem is similar to Problem 7.16. It is about the potential energy stored in a spring. We'd like to know how much the spring has to be stretched in order to store a given amount of energy.
DEVELOP The amount of energy stored in a spring is given by Equation 7.4, $U = kx^2/2$, where x is the distance stretched (or compressed) from its natural length.
EVALUATE Assume one starts stretching from the unstretched position (x = 0). Solving Equation 7.4 for x gives

$$x = \pm\sqrt{\frac{2U}{k}} = \pm\sqrt{\frac{2(210\text{ J})}{1400\text{ N/m}}} = \pm 55\text{ cm}$$

ASSESS The positive and negative signs indicate that you can store the same amount of energy by either compressing the spring or by stretching the spring.

18. **INTERPRET** We're asked to find the energy stored in the molecule as it is stretched.
DEVELOP The molecule acts like a spring, so its potential energy increases as it is pulled apart according to Equation 7.4: $U = \frac{1}{2}kx^2$.
EVALUATE Using the values given, the potential energy of the stretched molecule is

$$U = \tfrac{1}{2}(0.046\text{ pN}/\mu\text{m})(26\ \mu\text{m})^2 = 16\times\left(10^{-12}\text{ N}\right)\left(10^{-6}\text{ m}\right) = 1.6\times10^{-17}\text{ J}$$

ASSESS This is equivalent to 10 eV. The energy in molecular bonds is usually measured in eV, so the answer appears to be of the right magnitude. It's also positive because the stretched molecule has stored potential energy that will convert to kinetic energy (most likely vibration) when the molecule is released.

Section 7.3 Conservation of Mechanical Energy

19. **INTERPRET** This problem involves potential and kinetic energy. Because the slope is frictionless, the total mechanical energy is conserved, so $K + U$ = constant. We are interested in finding the speed of the skier after he descends each section of the slope.
DEVELOP We define the zero of potential energy at the top of the hill. Also, because the skier's speed there is zero, his initial kinetic energy is zero. Thus, his initial total mechanical energy is zero. Use Equation 7.3, $U = mg\Delta y$, to express his potential energy at the bottom of each slope, and Equation 6.13, $K = mv^2/2$, to express his kinetic energy at each location. Applying conservation of total mechanical energy to find the speed gives

$$0 = K + U = \frac{1}{2}mv^2 + mg\Delta y$$
$$v = \pm\sqrt{-2g\Delta y}$$

EVALUATE After the first slope, $\Delta y = -25$ m, so we have

$$v = \pm\sqrt{-2\left(9.8\text{ m/s}^2\right)\left(-25\text{ m}\right)} = \pm 22\text{ m/s}$$

After the second slope, we have

$$v = \pm\sqrt{-2\left(9.8\text{ m/s}^2\right)\left(-25\text{ m} - 38\text{ m}\right)} = \pm 35\text{ m/s}$$

ASSESS The plus/minus sign indicates that the result is independent of the direction in which he is skiing. It is the same whether he skis to the left or to the right on the level sections.

20. **INTERPRET** This problem involves the conservation of mechanical energy, since the force on the spring is conservative. The kinetic energy of the plane is therefore all converted into potential energy of the spring.

DEVELOP Using Equation 7.7, the sum of the kinetic energy of the plane and the potential energy of the spring is a constant: $K + U = \text{constant}$.

EVALUATE Initially, the plane is moving $\left(K_0 = \tfrac{1}{2}mv_0^2\right)$, and the spring is unstretched $\left(U_0 = 0\right)$. When the plane comes to a stop $\left(K = 0\right)$, the spring has stretched a certain distance and thereby gained potential energy $\left(U = \tfrac{1}{2}kx^2\right)$. Equating the initial and final mechanical energies $\left(K_0 + U_0 = K + U\right)$ allows us to solve for the landing speed:

$$v_0 = \sqrt{\frac{k}{m}}x = \sqrt{\frac{40\text{ kN/m}}{10{,}000\text{ kg}}}\left(25\text{ m}\right) = 50\text{ m/s} = 110\text{ mi/h}$$

ASSESS Although much slower than the cruising speed of a fighter jet $\left(\sim 700\text{ mi/h}\right)$, this is a typical landing speed. But note, the pilot typically throttles the engines when the plane touches down so that it can make a hasty take-off in case the tailhook misses the landing cable.

21. **INTERPRET** This problem involves the conservative forces of gravity and the elastic force, so we can apply the conservation of mechanical energy to this problem. We are interested in finding the height to which the arrow rises, given its initial elastic potential energy.

DEVELOP We will take the initial position of the arrow to be the zero of potential energy. The initial total mechanical energy of the arrow is then just the elastic potential energy of the arrow, $U_e = kx^2/2$, with $x = 0.71$ m and $k = 430$ N/m. The final total mechanical energy of the arrow is simply the gravitational potential energy, $U_g = mg\Delta y$, because the arrow has zero speed at the peak of its trajectory, so its kinetic energy there is zero.

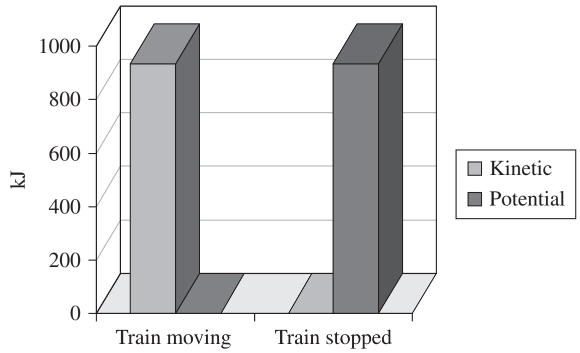
EVALUATE By conservation of total mechanical energy, we equate the initial and final total mechanical energies to find the height Δy to which the arrow rises. The result is

$$U_e = U_g$$
$$\frac{1}{2}kx^2 = mg\Delta y$$
$$\Delta y = \frac{kx^2}{2mg} = \frac{\left(430\text{ N/m}\right)\left(0.71\text{ m}\right)^2}{2\left(0.12\text{ kg}\right)\left(9.8\text{ m/s}^2\right)} = 92\text{ m/s}$$

ASSESS Notice that the height is measured from the arrow's position when the bow is taught, because that is the position at which the arrow has the elastic potential energy.

22. **INTERPRET** We are asked to find the amount a spring compresses as it stops a boxcar. This is a conservation of energy problem involving elastic potential energy and kinetic energy. The boxcar has kinetic energy before it hits the spring, and the spring has elastic potential energy when the boxcar is stopped.

DEVELOP Use conservation of total mechanical energy: $U_i + K_i = U_f + K_f$. The initial energy is entirely kinetic, and the final energy is entirely elastic potential energy (see bar chart, below). The elastic potential energy of the spring is $U_f = kx^2/2$, with $k = 1.8$ MN/m, and the kinetic energy of a moving train is $K_i = mv^2/2$, with $v = 7.5$ m/s and $m = 35{,}000$ kg.



EVALUATE By conservation of total mechanical energy, we have

$$\overbrace{U_i}^{=0} + K_i = U_f + \overbrace{K_f}^{=0}$$

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$x = \pm v\sqrt{\frac{m}{k}} = -(7.3\text{ m/s})\sqrt{\frac{35{,}000\text{ kg}}{1.8\text{ MN/m}}} = -1.0\text{ m}$$

where we have retained the negative sign to indicate that the spring compresses.

ASSESS Check the units: $\sqrt{\frac{m}{k}}$ has units of seconds, so $[m/s]\times[s]=[m]$ and we're fine. Keep your eyes on this term $\sqrt{\frac{k}{m}}$, though—it becomes very important in later chapters!

23. **INTERPRET** We are to find the spring constant needed to launch a toy rocket to a given height. We use the conservation of total mechanical energy: The initial energy is the elastic potential energy of the spring, and the final energy is gravitational potential energy.

DEVELOP Conservation of total mechanical energy says that $U_i + K_i = U_f + K_f$. For this problem, the initial and final kinetic energies are zero. From Equation 7.4, we know that the initial elastic potential energy of the spring is $U_i = kx^2/2$, and the final gravitational energy is $U_f = mgh$. The spring compression is $x = -0.14$ m, the rocket's mass is $m = 65$ g $= 0.065$ kg, and the desired height is $h = 35$ m.

EVALUATE Applying the conservation of total mechanical energy and solving for the spring constant k gives

$$U_i + \overbrace{K_i}^{=0} = U_f + \overbrace{K_f}^{=0}$$

$$\frac{1}{2}kx^2 = mgh$$

$$k = \frac{2mgh}{x^2} = \frac{2(0.065\text{ kg})(9.8\text{ m/s}^2)(35\text{ m})}{(0.14\text{ m})^2} = 2.3\text{ kN/m}$$

ASSESS This spring is probably a bit stiff for a kid's toy. It will take a force of 320 N to completely compress the spring, which is about 70 lbs.

Section 7.4 Potential-Energy Curves

24. **INTERPRET** The object of interest is the particle. As it slides along a frictionless track, energy is converted from gravitational potential energy to kinetic and vice versa, but the overall total mechanical energy is conserved at every point. Use this principle to calculate the speed and position of the particle at various points on the track.

DEVELOP Let the kinetic energy of the particle be $K = mv^2/2$ and the gravitational potential energy be $U = mgy$ (measured from the reference level $y = 0$ in Fig. 7.16). Because the track is frictionless, we can use the conservation of total mechanical energy stated in Equation 7.7, which is

$$K + U = \text{constant}$$

We are given that $v_A = 0$, so the initial kinetic energy is zero. The initial potential energy can be calculated using Equation 7.3, $U = mgy$, with $y_A = 3.8$ m. Thus, we can evaluate the total mechanical energy at point A and the total mechanical energy at any other point must have the same value:

$$U_i + \overbrace{K_i}^{=0} = U_f + K_f$$
$$mgy_A = mgy + \frac{1}{2}mv^2$$

EVALUATE **(a)** Applying the conservation of total mechanical energy to point B, we find

$$mgy_A = \frac{1}{2}mv_B^2 + mgy_B$$

Solving for the speed v_B, we find

$$v_B = \pm\sqrt{2g\left(y_A - y_B\right)} = \pm\sqrt{2\left(9.8\ \text{m/s}^2\right)\left(3.8\ \text{m} - 2.6\ \text{m}\right)} = \pm 4.9\ \text{m/s}$$

(b) Performing an analogous calculation for point C gives us

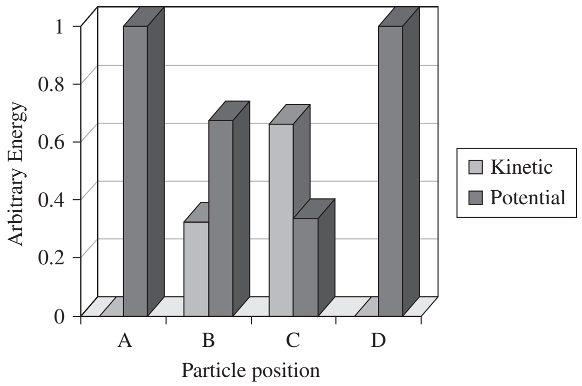
$$v_C = \pm\sqrt{2g(\left(y_A - y_C\right))} = \pm\sqrt{2\left(9.8\ \text{m/s}^2\right)\left(3.8\ \text{m} - 1.3\ \text{m}\right)} = \pm 7.0\ \text{m/s}$$

(c) The right-hand turning point is where the particle reverses direction. At this point, the velocity will be instantaneously zero, so the total mechanical energy will consist only of the gravitational potential energy. Therefore, the particle must be at the same height as point A. Inspecting Fig. 7.16 tells us that this point is at $x \approx 11$ m.

ASSESS By mechanical energy conservation,

$$K + U = \frac{1}{2}mv^2 + mgy = \text{constant}$$

we see that the speed of the particle is a maximum at $y = 0$. Similarly, when the speed of the particle is zero, y is at a maximum. The bar chart below also shows the exchange between kinetic and potential energy (note that the energy scale is not absolute because we are not given the particle's mass).



25. **INTERPRET** This problem involves conservative forces and conservation of total mechanical energy. At the maximum height of the particle, we know its kinetic energy is zero and its potential energy is maximum. We will define the zero of potential energy as the particle's lowest position, where its kinetic energy will be maximum. We are asked to find the turning point of the particle, which is the x position where the particle stops rising and begins to fall again.

DEVELOP The particle's trajectory is given by the formula $y = ax^2$, with $a = 0.92\ \text{m}^{-1}$. The particles potential energy is $U = mgy$, and its kinetic energy is $K = mv^2/2$. Conservation of total mechanical energy tells us that the sum of these two quantities is conserved, and we can find that constant because we are told that the maximum speed (i.e., maximum kinetic energy) is 8.5 m/s, which must occur at the point where the potential energy is minimum (i.e., $y = 0$). Thus, we have

$$\frac{1}{2}mv_{\max}^2 = K_{\max}$$

where K_{max} is constant and is the total mechanical energy, which is conserved. We can insert this into the general expression for total mechanical energy to find the turning point of the particle, because we know that the particle kinetic energy will be zero at the turning point.

EVALUATE The total mechanical energy is

$$K_{max} = U + K = mgy + \frac{1}{2}mv^2 = mg(ax^2) + \frac{1}{2}mv^2$$

At the turning point, v = 0, so we have

$$K_{max} = \frac{1}{2}mv_{max}^2 = mg\left(ax_{turn}^2\right)$$

$$x_{turn} = \pm v_{max}\sqrt{\frac{1}{2ga}} = \pm(8.5 \text{ m/s})\sqrt{\frac{1}{2\left(9.8 \text{ m/s}^2\right)\left(0.92 \text{ m}^{-1}\right)}} = \pm 2.0 \text{ m}$$

ASSESS The positive/negative sign means that there are two turning points: one at positive 2.0 m from the origin (i.e., to the right) and one at –2.0 m from the origin (i.e., to the left).

26. **INTERPRET** This problem is about finding the force acting on a particle, given its potential energy as a function of position. The motion of the particle is in one dimension.

DEVELOP The force on a particle in one-dimensional motion is the negative derivative of the potential energy: $F(x) = -dU/dx$, as discussed in deriving Equation 7.8. When the function $U(x)$ consists of line segments, this formula simplifies to $F(x) = -\Delta U/\Delta x$, which we can use to solve this problem.

EVALUATE Using $F(x) = -\Delta U/\Delta x$, we find that the force is: (**a**) F_a = – (3 J)/(1.5 m) = –2 N, (**b**) F_b = – (0 J)/(1 m) =0 N, (**c**) F_c = – (–4 J)/(0.5 m) = 8 N, (**d**) F_d = – (–1 J)/(1 m) = 1 N, (**e**) F_e = – (4 J)/(1 m) = –4 N, (**f**) F_a = – (0 J)/(1 m) = 0 N

ASSESS The result indicates that the greater is the change of the potential energy $U(x)$ with x, the greater is the force on the particle. Integrating Equation 7.8 leads to Equation 7.2a:

$$\Delta U(x) = -\int F(x)dx$$

But $\int F(x)dx$ is just the work W done by the force. Therefore, we can say that $W = -\Delta U$.

27. **INTERPRET** For this one-dimensional problem, we are given the potential energy of a particle as a function of the particle's position and are asked to find the force on the particle.

DEVELOP Because this problem is one-dimensional, we will use Equation 7.8, $F(x) = -dU/dx$, to find the force on the particle. The potential energy of the particle is $U(x) = 16x^2 - b$, with b = 4.0 J, so the force is $F(x) -dU/dx = -32x$.

EVALUATE (**a**) At x = 2.1 m, the force is $F(x$ = 2.1 m) = –32(2.1 m) = 67 N.

(**b**) At x = 0 m, the force is $F(x$ = 0 m) = –32(0 m) = 0.0 N.

(**c**) At x = –1.4 m, the force is $F(x$ = –1.4 m) = –32(–1.4 m) = 45 N.

ASSESS Notice that the results are given to two significant figures because the data is given to two significant figures.

PROBLEMS

28. **INTERPRET** Water is pumped to a higher reservoir to store potential energy. We need to calculate the gravitational potential energy of the reservoir, and the time it would take to drain the reservoir given the power output of the generators. Although it is not stated in the problem, we will assume that the efficiency of the generators is 100%.

DEVELOP The mass of the reservoir is $m = 2.1\times 10^{10}$ kg, and the height above the generators is $h = 214$ m. The initial gravitational potential energy is $U_0 = mgh$, since the level of the generators is taken to be $U = 0$. As the reservoir drains, there will be less water and therefore less potential energy. This loss in potential energy goes into kinetic energy of the water, which does work on the generators: $W = \Delta K = -\Delta U$. The maximum work possible corresponds to draining the whole reservoir of water (letting it all flow down to $U = 0$), which is equivalent to using up all the initial potential energy:

$$W_{\text{max}} = -\Delta U_{\text{total}} = -(0 - U_0) = U_0$$

If the power is constant, then the amount of work done is $W = P\Delta t$ (recall Equation 6.17), so the time to drain the whole reservoir is $\Delta t = U_0 / P$.

EVALUATE (a) The total potential energy of the reservoir is

$$U_0 = mgh = (2.1\times10^{10}\ \text{kg})(9.8\ \text{m/s}^2)(214\ \text{m}) = 4.4\times10^{13}\ \text{J}$$

(b) The total time the generators can run before the reservoir is empty can be found from the equation above:

$$\Delta t = \frac{U_0}{P} = \frac{4.4\times10^{13}\ \text{J.}}{1.08\ \text{GW}} = 11\ \text{h}$$

ASSESS The energy stored in the full reservoir is equivalent to about 12 million kWh of electricity, or 12 GWh. As explained in the text, reservoirs such as this are often used to store power that can be used during periods of peak demand. One can imagine this facility generating power during the daylight hours, and then "recharging" (pumping water back up to the top reservoir) during the night.

29. **INTERPRET** This problem asks us to calculate the work done around a square path given two different force fields. If the forces are conservative, the work should be zero.

DEVELOP The work done by the given forces as an object moves around the box can be divided into 4 segments. Starting from the bottom left-hand corner, $d\vec{r} = \hat{i}dx,\ y = 0$ and x goes from 0 to a. Then, $d\vec{r} = \hat{j}dy,\ x = a$ and y goes from 0 to a. After, $d\vec{r} = \hat{i}dx,\ y = a$ and x goes from a to 0. And finally, $d\vec{r} = \hat{j}dy,\ x = 0$ and y goes from a to 0. Mathematically, we can write this as:

$$W = \oint \vec{F}\cdot d\vec{r} = \int_0^a \vec{F}\cdot\hat{i}dx\Big|_{y=0} + \int_0^a \vec{F}\cdot\hat{j}dy\Big|_{x=a} + \int_a^0 \vec{F}\cdot\hat{i}dx\Big|_{y=a} + \int_a^0 \vec{F}\cdot\hat{j}dy\Big|_{x=0}$$

In both cases, the force points in y-direction, so $\vec{F}\cdot\hat{i} = 0.$ Recall that we can reverse the endpoints of a definite integral by adding a minus sign: $\int_a^0 dx = -\int_0^a dx,$ so the work equation reduces to:

$$W = \oint \vec{F}\cdot d\vec{r} = \int_0^a \vec{F}\cdot\hat{j}dy\Big|_{x=a} - \int_0^a \vec{F}\cdot\hat{j}dy\Big|_{x=0}$$

EVALUATE For the constant force, $\vec{F}_a = F_0\hat{j}$, in Figure 7.14a, the work is

$$W_a = F_0\int_0^a dy - F_0\int_0^a dy = F_0 a - F_0 a = 0$$

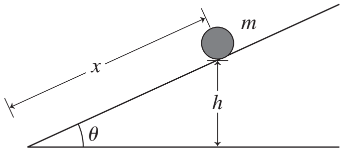
For the varying force, $\vec{F}_b = F_0(x/a)\hat{j},$ in Figure 7.14b, the work is

$$W_b = F_0\int_0^a (a/a)\,dy - F_0\int_0^a (0/a)\,dy = F_0 a - 0 = F_0 a$$

ASSESS The force $\vec{F}_a$ is conservative, but the force $\vec{F}_b$ is not. As far as the work done is concerned, it does matter what path an object takes when moving through the force field in Figure 7.14b.

30. **INTERPRET** In this problem we are asked to find the gravitational potential energy of a mass on an incline. As the mass moves up and down the incline, its potential energy changes because its height varies. We are requested to take the bottom of the incline (i.e., $x = 0$) as the zero of potential energy.

DEVELOP Draw a diagram of the situation (see figure below). From Equation 7.3, we know the change in gravitational potential energy is $\Delta U = mg\Delta y$. The delta symbols mean we consider the change in the corresponding variable, so $\Delta U = U_{\text{final}} - U_{\text{initial}}$ and $\Delta y = y_{\text{final}} - y_{\text{initial}}$. We are given that the zero of the potential energy is at the bottom of the incline, so $U_{\text{initial}} = 0$ at $y_{\text{initial}} = 0$ and Equation 7.3 reduces to $U(x) = mgy(x)$. From the geometry of the situation, we know $y(x) = x\sin(\theta)$.



EVALUATE Using the expression for the mass's height in the expression for potential energy, we find

$$U(x) = mgy(x) = mgx\sin(\theta)$$

ASSESS To see that our result makes sense, we can check the following limits: (i) $\theta = 0$; this situation corresponds to a flat surface, and $U = 0$ because $x = 0$. (ii) $\theta = 90°$; this corresponds to a vertical incline. The potential energy is simply $U(x) = mgx$ because x is the vertical height in this case.

31. **INTERPRET** This problem involves finding the gravitational potential energy of a brick that is placed in various different positions. We can treat the brick as if all its mass were located at the center of the brick. We are instructed to take the zero of the gravitational potential energy to be at the center of the brick when the brick is lying on its longest side.

DEVELOP Draw a diagram of the brick when it is lying on its longest side, and when it is in the positions for parts (a) and (b) of the problem (see figure below). The gravitational potential energy of the brick is given by Equation 7.3:

$$\Delta U = mg\Delta y = mg(y - y_0)$$

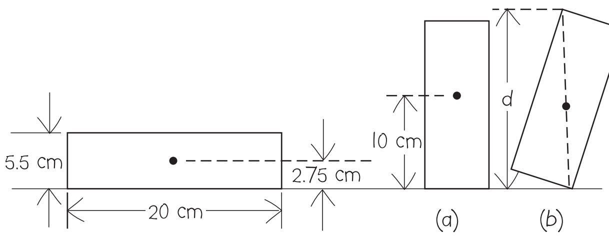
where Δy is the vertical distance of the center of the brick above the point of zero of potential energy, which is y_0. From the figure below, we see that $y_0 = 2.75$ cm.

In position (a) the change in height of the center of the brick is

$$\Delta y_a = y_a - y_0 = 10.0 \text{ cm} - 2.75 \text{ cm} = 7.25 \text{ cm}$$

In (b), the change in height of the brick is

$$\Delta y_b = \tfrac{1}{2}\sqrt{(20.0 \text{ cm})^2 + (5.50 \text{ cm})^2} - 2.75 \text{ cm} = 10.4 \text{ cm} - 2.75 \text{ cm} = 7.62 \text{ cm}.$$



EVALUATE From Equation 7.3, the gravitational potential energies at positions **(a)** and **(b)** are

$$U_a = mg\Delta y_a = (1.50 \text{ kg})(9.82 \text{ m/s}^2)(0.0725 \text{ m}) = 1.07 \text{ J}$$

$$U_b = (1.50 \text{ kg})(9.82 \text{ m/s}^2)(0.0762 \text{ m}) = 1.12 \text{ J}$$

ASSESS The potential energy is larger for part (b) because the center of the brick is higher than it is for part (a). Notice that we had to use the acceleration of gravity to three significant figures for this problem. Had we used $g =$ 9.8 m/s2, our results would be $U_a = U_b = 1.1$ J.

32. **INTERPRET** This problem involves elastic potential energy. The object of interest is the molecular bond between the oxygen and the carbon atoms, and we are to find the effective spring constant of this bond.

DEVELOP The zero of the potential energy is at the equilibrium position of the atoms. The elastic potential energy is given by Equation 7.4, $U = kx^2/2$.

EVALUATE Inserting the given quantities into the expression for elastic potential energy gives

$$U = \frac{1}{2}kx^2 \quad \Rightarrow \quad k = \frac{2U}{x^2} = \frac{2(0.015 \text{ eV})}{(1.6\times10^{-12} \text{ m})^2}\left(\frac{1.602\times10^{-19} \text{ J}}{\text{eV}}\right) = 1.9 \text{ kN/m}$$

where we have used the conversion factor from eV to J from Appendix C.

Assess This spring constant is two orders of magnitude greater than that of a bungee cord, which is around 10 N/m.

33. **Interpret** This problem deals with the elastic potential energy stored in a rope. We need to find the elastic potential energy stored in a rope stretched the given amount and compare the result with that of Example 7.3, which uses a different expression for the force exerted by the rope.

Develop The force exerted by the rope is $F(x) = -kx + bx^2 - cx^3$, where x is the length the rope is stretched from its equilibrium position. Use Equation 7.2a, $\Delta U = -\int_{x_1}^{x_2} F(x)\,dx$, to find the elastic potential energy of the rope.

Evaluate Performing the required integration and inserting the known quantities, we obtain

$$U = -\int_0^x F(x')\,dx' = -\int_0^x \left(-kx' + bx'^2 - cx'^3\right)dx' = \left(\frac{1}{2}kx'^2 - \frac{1}{3}bx'^3 + \frac{1}{4}cx'^4\right)_{x'=0}^{x'=2.62\text{ cm}}$$

$$= \frac{1}{2}(223\text{ N/m})(2.62\text{ m})^2 - \frac{1}{3}\left(4.10\text{ N/m}^2\right)(2.62\text{ m})^3 + \frac{1}{4}\left(3.1\text{ N/m}^3\right)(2.62\text{ m})^4$$

$$= 778\text{ J}$$

In Example 7.3, the energy stored is $U' = 741$ J. Therefore, the percent difference is

$$(100\%)\frac{U - U'}{U'} = (100\%)\frac{778\text{ J} - 741\text{ J}}{741\text{ J}} = 4.90\%$$

Assess Adding the term $-cx^3$ increases the potential energy of the system. The negative sign increases the restoring force, and thus the work needed to stretch the spring.

34. **Interpret** We're asked to characterize the Achilles tendon as a mechanical spring.
Develop When a 125-kg mass is hung on the tendon, it stretched until its restorative spring force countered the mass' weight: $kx = mg$. From this, we can find the spring constant. And furthermore, we can use Equation 7.4, $U = \frac{1}{2}kx^2$, to find the distance the tendon must stretch in order to store 50.0 J of energy.

Evaluate (a) The experiment with the mass is enough to tell us what the spring constant is for the tendon:

$$k = \frac{mg}{x} = \frac{(125\text{ kg})\left(9.8\text{ m/s}^2\right)}{2.66\text{ mm}} = 4.605\times10^5\text{ N/m} \simeq 461\text{ N/mm}$$

(b) In order to store 50.0 J in the tendon, it must be stretched by

$$x = \sqrt{\frac{2U}{k}} = \sqrt{\frac{2(50.0\text{ J})}{\left(4.605\times10^5\text{ N/m}\right)}} = 14.7\text{ mm}$$

Note that we put k in units of N/m in order to avoid confusion in the units, since $\text{J/(N/m)} = \text{m}^2$.
Assess In general, tendons connect muscle to bone. The Achilles tendon, in particular, connects the muscles in the lower leg to the heel bone. It therefore has to withstand forces equal to and greater than that of a person's body weight without stretching too much. The relatively large spring constant that we found in part (a) bears witness to that fact.

35. **Interpret** We want to find the energy stored in an unusual spring when it's compressed a certain distance.
Develop The energy stored in the spring is just the potential energy. For a one-dimensional force and displacement, we can use Equation 7.2a:

$$\Delta U = -\int_0^x F(x')dx' = -\int_0^x \left(-kx' - cx'^3\right)dx' = \left[\tfrac{1}{2}kx'^2 + \tfrac{1}{4}cx'^4\right]_0^x = \tfrac{1}{2}kx^2 + \tfrac{1}{4}cx^4$$

Evaluate In this case the spring is compressed, which we will define as being in the negative x-direction. The stored energy is therefore:

$$\Delta U = \tfrac{1}{2}(220\text{ N/m})(-0.15\text{ m})^2 + \tfrac{1}{4}\left(3.1\text{ N/m}^3\right)(-0.15\text{ m})^4 = 2.5\text{ J}$$

Assess The units come out the same (i.e., $\text{N}\cdot\text{m}$) for both terms in the sum, as they should. Notice that for the given force equation, it makes no difference whether the spring is compressed or stretched – the stored energy is the same for the same displacement.

36. **Interpret** This one-dimensional problem is about finding the potential-energy difference of a particle between two positions when the force acting on a particle is known.

Develop According to Equation 7.2, when an object moves from x_1 to x_2 under the influence of a force $\vec{F}(x) = (Ax^{-2})\hat{i}$, the change in potential energy is

$$\Delta U = -\int_{x_1}^{x_2} F(x)\,dx$$

Evaluate **(a)** Performing the integration and inserting the given quantities, we obtain

$$U(x_2) - U(x_1) = -\int_{x_1}^{x_2} Ax^{-2}dx = \left.\frac{A}{x}\right|_{x_1}^{x_2} = A\left(\frac{1}{x_2} - \frac{1}{x_1}\right)$$

(b) For $x_1 \to \infty$, $U(x_2) - U(\infty) = A/x_2$. Thus, the potential energy remains finite in this limit. In this case, it makes sense to define the zero of potential energy at infinity, $U(\infty) = 0$, so $U(x) = A/x$.

Assess The negative sign in $\Delta U = -\int_{x_1}^{x_2} F(x)\,dx$ means that if the work done by the force is positive, then the potential energy must decrease.

37. **Interpret** This problem is similar to the preceding problem. We are given a force as a function of position, and we are to find the change in potential energy. For this problem, we are given the zero of the potential energy, so we will find the potential energy with respect to this zero.

Develop Use Equation 7.2(a),

$$\Delta U = -\int_{x_1}^{x_2} F(x)\,dx$$

to find the force. Because U(x = 0) = 0, this expression reduces to

$$\Delta U = U(x_2) - \overbrace{U(x_1 = 0)}^{=0} \equiv U(x) - \int_0^x F(x')\,dx'$$

Evaluate Performing the integration gives

$$U(x) = -\int_0^x F(x')\,dx' = -\int_0^x \left(ax'^2 + b\right)dx' = -\frac{1}{3}ax^3 - bx$$

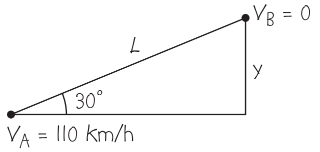
Assess The potential energy decreases with *x*, meaning that the force does ever increasing work on the particle as it moves.

38. **Interpret** The problem is about conservation of mechanical energy. Both potential energy and kinetic energy are involved. The kinetic energy of the truck is converted to gravitational potential energy as it moves uphill.

Develop Initially the kinetic energy of the truck is $K_0 = \frac{1}{2}mv_A^2$, and you can set its gravitational potential energy as $U_0 = 0$. In the final state, all the kinetic energy has been converted to gravitational potential energy: $K = 0$ and $U = mgy$. These quantities are related by the principle of conservation of mechanical energy given in Equation 7.7: $K_0 + U_0 = K + U$, which gives you:

$$\tfrac{1}{2}mv_A^2 + 0 = 0 + mgy$$

You are asked to find how long the lane should be in order to stop a runaway truck going $v_A = 110$ km/h $= 30.6$ m/s. The length of the lane, *L*, is related to the height by: $L = y/\sin\theta$, see the figure below.



Evaluate Solving for the length, you have:

$$L=\frac{v_{\mathrm{A}}^2}{2g\sin\theta}=\frac{(30.6\ \mathrm{m/s})^2}{2(9.8\ \mathrm{m/s^2})\sin 30°}=95\ \mathrm{m}$$

ASSESS Notice that the mass of the truck is irrelevant. The distance the runaway truck travels only depends on its velocity and the angle of the incline. If you make the incline steeper, the truck will still climb to the same height, y, but the length of the lane can be made shorter.

39. **INTERPRET** This problem involves conservative forces, that due to the spring and that due to gravity. Therefore, assuming the slope is frictionless, conservation of mechanical energy applies so we know that the total mechanical energy at all points on the trajectory of the block is a constant. Without loss of generality, we can take the zero of the gravitational potential energy to be at the position of the block before the spring is released.
DEVELOP Initially, before the spring is released, the total mechanical energy is the elastic potential energy of the spring, $U_{\mathrm{Tot}} = kx^2/2$. At the highest point reached by the block, its speed is instantaneously zero, so the final total mechanical energy is its gravitational potential energy, and $U_{\mathrm{Tot}} = mg\Delta y$, where Δy is the height above the starting point. By trigonometry, this height is $\Delta y = r\sin(\theta)$, where r is the distance the block travels up the incline.
EVALUATE By conservation of total mechanical energy, we equate the two expressions above for total mechanical energy. Inserting the expression Δy and solving for r gives

$$\frac{kx^2}{2}=mg\Delta y=mgr\sin(\theta)$$
$$r=\frac{kx^2}{2mg\sin(\theta)}$$

ASSESS The distance traveled up the incline is quadratic in x (the spring compression), so if we compress the spring twice as much, the mass will travel 4 times the distance.

40. **INTERPRET** The problem is about conservation of mechanical energy. Both kinetic energy and gravitational potential energy are involved. The object of interest is the child on the swing. As the swing moves, energy is exchanged between kinetic and potential energy, but the total mechanical energy remains constant. We will take the zero of gravitational potential energy to be the lowest point of the swing.
DEVELOP Let L denote the length of the chain of the swing. Suppose the swing is released from an angle θ_0, which is measured with respect to the vertical. This position corresponds to a vertical height of $y_0 = \mathrm{L}(1 - \cos\theta_0)$. After the child is released, the swing attains a maximum speed at the lowest point, where all the gravitational potential energy is converted to kinetic energy. Conservation of mechanical energy (Equation 7.7) gives

$$K_0+U_0=K+U=\text{constant}$$

from which the maximum speed can be calculated.
EVALUATE Equation 7.7 implies that

$$\overbrace{0}^{K_0}+\overbrace{mgy_0}^{U_0}=\overbrace{\frac{1}{2}mv_{\max}^2}^{K}+\overbrace{0}^{U}$$
$$mgL(1-\cos\theta_0)=\frac{1}{2}mv_{\max}^2$$

This gives

$$v_{\max}=\pm\sqrt{2gL(1-\cos\theta)_0}=\pm\sqrt{2(9.8\ \mathrm{m/s^2})(3.2\ \mathrm{m})[1-\cos(50°)]}=\pm 4.7\ \mathrm{m/s}$$

ASSESS The result shows that, as every kid knows, increasing the initial angle θ_0 gives a greater speed at the bottom of the swing. The positive and negative sign indicates that the swing may descend right-to-left or left-to-right, or that it does not matter to which side of the minimum we initially raise the swing.

41. **INTERPRET** Ignoring air resistance, the only force acting on the object once its release is gravity, which is a conservative force. Thus, this problem involves conservation of total mechanical energy. We will take the final position of the object to be the zero of the gravitational potential energy. Our goal is to derive Equation 2.11 by applying conservation of total mechanical energy.

DEVELOP The instant the ball is released, its total mechanical energy is $K_0 + U_0 = m v_0^2/2 + mgh$, where h is the height above the zero of the potential energy. Without loss of generality, we take the final position to be the zero of the potential energy, so the final mechanical energy is then is $K + U = mv^2/2$. By conservation of total mechanical energy, these two expressions give the same result, so we can equate them and solve for the final speed v.

EVALUATE The final speed of the object is

$$\frac{mv_0^2}{2} + mgy_0 = \frac{mv^2}{2}$$
$$\frac{v^2}{2} = \frac{v_0^2}{2} + gh$$
$$v = \pm\sqrt{v_0^2 + 2gh} = -\sqrt{v_0^2 + 2gh}$$

where we have taken the negative square root because the object is traveling in the negative direction (i.e., from large h to small h).

ASSESS Notice that the formula given in the problem statement should have a ± sign because the square root is involved.

42. **INTERPRET** To find the energy stored in the ligament, we integrate the force equation with respect to the distance, x. The force is one-dimensional so we don't need to worry about vectors, but we will have to be careful about signs.

DEVELOP The force in the graph, $F(x)$, is the force applied to the ligament in order to stretch it a given distance. By Newton's third law, the force of the ligament is equal and opposite to this applied force: $F_{\text{lig}} = -F$. To find the energy stored in the ligament, we integrate the ligament's force according to Equation 7.2a:

$$\Delta U = -\int_0^x F_{\text{lig}}(x')dx' = \int_0^x F(x')dx' = \tfrac{1}{2}0.43x^2 - \tfrac{1}{3}0.033x^3 + \tfrac{1}{4}0.00086x^4$$

where the units are $\text{kN}\cdot\text{cm} = 10\ \text{J}$, since F is in kN and x is in cm.

EVALUATE (a) For $x = 7.5\ \text{cm}$:

$$\Delta U = \tfrac{1}{2}0.43(7.5)^2 - \tfrac{1}{3}0.033(7.5)^3 + \tfrac{1}{4}0.00086(7.5)^4 = 8.1\ \text{J}$$

(a) For $x = 15\ \text{cm}$:

$$\Delta U = \tfrac{1}{2}0.43(15)^2 - \tfrac{1}{3}0.033(15)^3 + \tfrac{1}{4}0.00086(15)^4 = 22\ \text{J}$$

ASSESS The stored energy is the area under the curve out to the distance specified. As we would expect, the stored energy increases as the stretch distance increases.

43. **INTERPRET** The two forces acting on the block are those applied by the springs, so they are conservative forces. In the absence of friction and air resistance, we can apply conservation of total mechanical energy.

DEVELOP When the left hand spring is at its maximum compression, the block is instantaneously motionless, so the total mechanical energy of the block/springs system is just the elastic potential energy of the left-hand spring, so $U_{\text{L}}^{\text{Tot}} = k_{\text{L}}x_{\text{L}}^2/2$. At the opposite end, the total mechanical energy is $U_{\text{R}}^{\text{Tot}} = k_{\text{R}}x_{\text{R}}^2/2$. Between the springs the total energy is just the kinetic energy, so $U_{\text{K}}^{\text{Tot}} = mv^2/2$. By conservation of total mechanical energy, we can equate all three energies to find the compression x_{R} of the right-hand spring and the speed v of the block between the springs.

EVALUATE (**a**) At the right-hand end, the spring compresses a distance

$$k_L x_L^2 = k_R x_R^2$$
$$x_R = \pm x_L\sqrt{\frac{k_L}{k_R}} = -(0.16\ \text{m})\sqrt{\frac{130\ \text{N/m}}{280\ \text{N/m}}} = -11\ \text{cm}$$

where we have chosen the negative sign because the right-hand spring compresses.

(**b**) The speed of the block between the springs is

$$\frac{k_L x_L^2}{2} = \frac{mv^2}{2}$$

$$v = \pm x_L \sqrt{\frac{k_L}{m}} = \pm(0.16\ \text{m})\sqrt{\frac{130\ \text{N/m}}{0.2\ \text{kg}}} = \pm 4\ \text{m/s}$$

where we have kept both signs because the block can move either left-to-right or right-to-left, and we have retained only a single significant figure because we only know the block's weight to a single significant figure.

ASSESS For part (b), the units of the right-hand side are

$$\text{m}\sqrt{\frac{\text{m}}{\text{kg}}} = \text{m}\sqrt{\frac{\text{kg}\cdot\text{m}\cdot\text{s}^{-2}/\text{m}}{\text{kg}}} = \text{m/s}$$

as expected.

44. **INTERPRET** The bumper absorbs the kinetic energy of the car by transforming it into elastic potential energy. You can calculate the initial speed of the car by equating the two energies.

DEVELOP For the maximum collision speed, the car comes in with a kinetic energy of $K = \frac{1}{2}mv_{\text{max}}^2$. This energy is used to compress the spring until all the kinetic energy is converted to potential energy $U = \frac{1}{2}kx_{\text{max}}^2$.

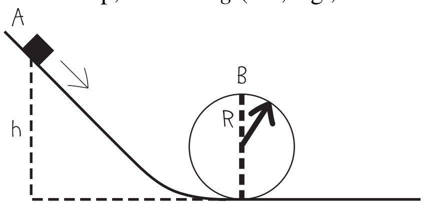
EVALUATE Equating the energies and solving for the maximum collision speed gives

$$v_{\text{max}} = \sqrt{\frac{k}{m}}x = \sqrt{\frac{1.3\ \text{MN/m}}{1400\ \text{kg}}}(5.0\ \text{cm}) = 5.5\ \text{km/h}$$

ASSESS At higher speeds, bumpers (and other parts of the car) are designed to crumple. This absorbs some of the kinetic energy, so as to reduce the shock on the passengers. But the crumpling cannot be undone like the spring compression.

45. **INTERPRET** Because the track is frictionless (and we ignore air resistance), the only force acting on the block is gravity, which is a conservative force. Therefore, we can apply the conservation of total mechanical energy to this problem. We will choose the zero of gravitational potential energy to be the base of the loop.

DEVELOP Apply conservation of total mechanical energy, Equation 7.7 ($U_0 + K_0 = U + K$). The initial total mechanical energy is just the gravitational potential energy because the speed (i.e., kinetic energy) is zero at the start. Therefore, $U_0 = mgh$. The energy at the top of the loop is $U + K = 2mgR + mv^2/2$, where R is the radius of the loop (see figure below). For the block to stay on the track, the centripetal acceleration of the block must exceed the acceleration due to gravity at the top of the loop, or $v^2/R \geq g$ (see, e.g., Problems 5.39 or 5.42).



EVALUATE Equating the two expressions for total mechanical energy and using the minimum speed criterion gives

$$mgh = 2mgR + \frac{1}{2}mv^2$$

$$v^2 = 2gh - 4gR \geq gR$$

$$h \geq \frac{5}{2}R$$

ASSESS Because real tracks always have some friction, the actual height needed would be greater than $5R/2$.

46. **INTERPRET** The problem is about conservation of total mechanical energy. The object of interest is the pendulum. As it swings, energy is converted from kinetic energy to gravitational potential energy and vice versa, but the total mechanical energy does not change. We choose the zero of gravitational potential energy to be the lowest point of the pendulum's swing.

DEVELOP Let L denote the length of the pendulum and let the pendulum be released from an angle θ_0 measured with respect to the vertical. This position corresponds to a height of $y_0 = L(1 - \cos\theta_0)$. At this point its speed is zero (i.e. kinetic energy is zero), so the initial total energy is $U_0 + K_0 = mgy_0$. The pendulum attains a maximum speed at the lowest point of the swing, where all the gravitational potential energy has been converted to kinetic energy, so the total mechanical energy here is $U + K = mv^2/2$. Apply conservation of total mechanical energy (Equation 7.7) to find the pendulum's length.

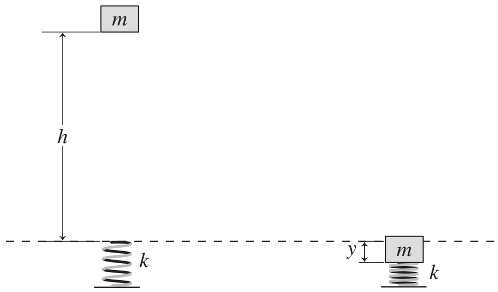
EVALUATE Conservation of total mechanical energy gives

$$U_0 + \overbrace{K_0}^{=0} = \overbrace{U}^{=0} + K$$
$$mgy_0 = \frac{1}{2}mv_{\max}^2$$
$$mgL\left(1-\cos\theta_0\right) = \frac{1}{2}mv_{\max}^2$$
$$L = \frac{v_{\max}^2}{2g\left(1-\cos\theta_0\right)} = \frac{\left(0.55\text{ m/s}\right)^2}{2\left(9.8\text{ m/s}^2\right)\left[1-\cos\left(8.0^\circ\right)\right]} = 1.6\text{ m}$$

ASSESS The result shows that L is inversely proportional to $1 - \cos\theta_0$. This means that if the maximum speed at the bottom of the swing is to remain unchanged, then increasing the initial angle θ_0 must be accompanied by a decrease in L.

47. **INTERPRET** This problem involves the forces of gravity and of an elastic spring, both of which are conservative forces. Therefore, we can apply conservation of total mechanical energy. We take the zero of the gravitational potential energy the height of the spring in equilibrium.

DEVELOP To apply conservation of total mechanical energy (Equation 7.7), we need to express the total mechanical energy for the block before it is dropped and when the spring is maximally compressed by the block (at which point the block is instantaneously motionless, see figure below). For the former, we have $U_0 + K_0 = mgh$. For the latter, we have $U + K = ky^2/2 - mgy$, where y is the distance from equilibrium that the spring is compressed.



EVALUATE Equating the two expressions above for total mechanical energy and solving for the maximum spring compression y gives

$$mgh = \frac{1}{2}ky^2 - mgy$$
$$\left(\frac{k}{2}\right)y^2 + \left(-mg\right)y + \left(-mgh\right) = 0$$
$$y = \frac{mg \pm \sqrt{m^2g^2 + 2kmgh}}{k} = \frac{mg}{k}\left(1 + \sqrt{1 + 2kh/mg}\right)$$

ASSESS We have retained the positive sign because the spring would not be compressed if $y < 0$.

48. **INTERPRET** This problem involves conservation of total mechanical energy (which we are given). We are given the potential energy of a particle as a function of its position, and we are to find the particle's turning points (i.e., where it reverses course from leftward to rightward).

DEVELOP At the turning points, the particle is instantaneously motionless, so its kinetic energy is zero. Therefore, its total mechanical energy of $E = 3.5$ J is entirely in the potential energy, which is $U(x) = 7.0 - 8.0x + 1.7x^2$.

EVALUATE Equating the total mechanical energy to the potential energy, we find

$$3.5 = 7.0 - 8.0x + 1.7x^2$$
$$1.7x^2 - 8.0x + 3.5 = 0$$
$$x = \frac{8.0\text{ J/m} \pm \sqrt{(8.0\text{ J/m})^2 - 4(1.7\text{ J/m}^2)(3.5\text{ J})}}{2(1.7\text{ J/m}^2)}\text{ m} = 4.22\text{ m and }0.488\text{ m}$$

ASSESS A turning point is a point where the particle reverses its motion. One may readily verify that $U(4.2\text{ m}) = U(0.49\text{ m}) = 3.5$ J.

49. **INTERPRET** This is a one-dimensional problem in which we are to derive an expression for the potential energy given the force acting on a object as a function of position.

DEVELOP The force is conservative because it is a function of position (if we come back to the same position, we experience the same force). We can therefore apply Equation 7.2, which for one dimension reduces to Equation 7.2a,

$$\Delta U = U(x_2) - U(x_1) = -\int_{x_1}^{x_2} F(x)\,dx$$

For part (b), note that the total mechanical energy at the turning points is just the potential energy because the kinetic energy is zero at these points (object is reversing direction). Thus, find the points on the graph where the potential energy is equivalent to the total mechanical energy, which is given to be –1 J.

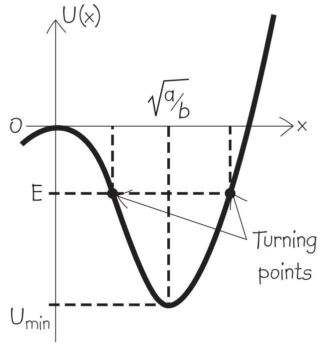
EVALUATE (**a**) Inserting the expression for force and performing the integration gives

$$\Delta U = -\int_{x_1}^{x_2} (ax - bx^3)\,dx = \left(\frac{ax^2}{2} - \frac{bx^4}{4}\right)\Bigg|_{x_1}^{x_2} = -\frac{a}{2}(x_2^2 - x_1^2) + \frac{b}{4}(x_2^4 - x_1^4)$$

Without loss of generality, we can define the potential energy at x = 0 to be zero, so

$$U(x) = -\frac{a}{2}x^2 + \frac{b}{4}x^4$$

(**b**) A graph of $U(x)$ for $x \ge 0$, when $a = 5$ N/m, $b = 2\text{ N/m}^3$, and x is in meters, is shown. (Note that the potential energy is symmetric, $U(-x) = U(x)$, but that only positive displacements are considered in this problem.) The conservation of energy can be written in terms of the total energy, $E = \frac{1}{2}m(dx/dt)^2 + U(x)$, so that $dx/dt = \pm\sqrt{2[E - U(x)]/m}$. The maximum speed occurs when $U(x)$ is a minimum; $dU/dx = 0$ and $d^2U/dx^2 > 0$. Taking the derivative, one finds $0 = -ax + bx^3$, which has solutions $x = 0$ and $x = \pm\sqrt{a/b} = \pm\sqrt{5/2}\text{ m} = \pm 1.58\text{ m}$. The second derivative $d^2U/dx^2 = -a + 3bx^2$ is negative for $x = 0$, which is a local maximum, but is positive for $x = \pm\sqrt{a/b}$, which are minima with $U_{\min} = U(\pm\sqrt{a/b}) = -a^2/4b =$ $-(25/8)\text{ J} = -3.13\text{ J}$. There is real physical motion $(K \ge 0)$ for total energy $E \ge U_{\min}$. The turning points (where $dx/dt = 0$) can be found from the equation $U(x) = E$; there are four solutions (two positive) for energies with $U_{\min} < E < 0$, and two solutions (one positive) for $E > 0$. The equation $U(x) - E = 0$ is equivalent to $x^4 - 2(a/b)x^2 - 4(E/b) = 0$. The quadratic formula gives $x = \pm\{(a/b) \pm [(a/b)^2 + 4(E/b)]^{1/2}\}^{1/2}$ for $U_{\min} < E < 0$, and $x = \pm\{(a/b) + [(a/b)^2 + 4(E/b)]^{1/2}\}^{1/2}$ for $E > 0$. For the particular values given $(E = -1\text{ J})$, the positive turning points are $x = \left[\left(5 \pm \sqrt{17}\right)/(2\text{ m})\right]^{1/2} = 0.662\text{ m}$ and 2.14 m, as can be seen in the graph below. Retaining one a single significant figure gives x = 0.7 and 2 m.



ASSESS The graph is identical for negative displacements.

50. INTERPRET In this problem we want to find the equilibrium separation between NaCl ions, given the potential energy function, $U(x)$.

DEVELOP At the equilibrium separation of the two ions, the potential energy is a minimum (see Figure 7.11). At a minimum, the derivative with respect to the separation is zero:

$$\frac{dU(r)}{dr} = 0$$

By solving the above equation, the equilibrium separation between ions in NaCl can be found.

EVALUATE Differentiating $U(r)$ with respect to r gives

$$\frac{dU}{dr} = 0 = -nbr_{\text{eq}}^{-(n+1)} + ar_{\text{eq}}^{-2}$$

Then solving for the equilibrium separation:

$$r_{\text{eq}} = \left(\frac{nb}{a}\right)^{1/(n-1)} = \left(\frac{(8.22)(5.52\times10^{-98})}{4.04\times10^{-28}}\right)^{1/(8.22-1)} = 2.82\times10^{-10}\,\text{m} = 2.82\ \text{Å}$$

where we have used the angstrom $(1\ \text{Å} = 10^{-10}\,\text{m})$, which is a common non-SI unit of length in chemistry and atomic physics.

ASSESS We can roughly estimate how far apart the ions are in table salt. The atomic masses of Na and Cl are 23.99 u and 35.45 u, respectively, where $1\ \text{u} = 1.661\times10^{-27}\,\text{kg}$. Let's assume that there's one Na-Cl pair inside a sphere radius r_{eq}, which means the "density" of one Na-Cl pair is roughly: $(m_{\text{Na}} + m_{\text{Cl}})/\left(\frac{4\pi}{3}r_{\text{eq}}^3\right)$. We can match this to the density of table salt $\left(\rho_{\text{NaCl}} = 2.17\ \text{g/cm}^3\right)$ to obtain an estimate for the equilibrium separation:

$$r_{\text{eq}} \sim \sqrt[3]{\frac{m_{\text{Na}} + m_{\text{Cl}}}{\frac{4\pi}{3}\rho_{\text{NaCl}}}} = \sqrt[3]{\frac{(23.99\ \text{u}) + (35.45\ \text{u})}{\frac{4\pi}{3}(2.17\ \text{g/cm}^3)}\left(\frac{1.661\times10^{-27}\,\text{kg}}{1\ \text{u}}\right)} = 2.2\ \text{Å}$$

This is approximately what we found above using the potential energy equation.

51. INTERPRET In this problem we are asked to find the speed of the skier at two different locations, given that the downward slope has a coefficient of friction $\mu_k = 0.11$. Because friction is a nonconservative force, we cannot apply conservation of total mechanical energy. Instead, we must use the concept of work done by a force combined with total mechanical energy.

DEVELOP We find the work done by the friction force and subtract this work from the total energy to find the energy remaining after each slope The work done by friction skiing down a straight slope of length L is

$$W_f = -f_k L = -\mu_k nL = -\mu_k\,(mg\cos\theta)\left(\frac{h}{\sin\theta}\right) = -\mu_k mgh\cot\theta$$

where $h = L\sin\theta$ is the vertical drop of the slope. Conservation of energy applied between the start and the first level section now gives $\Delta K_{AB} + \Delta U_{AB} = W_{f,AB}$ or

$$\frac{1}{2}mv_B^2 = mg(y_A - y_B) - \mu_k mg(y_A - y_B)\cot\theta_{AB}$$

Similarly, for the motion between the top and the second level, we must include all the work done by friction, so

$$\Delta K_{AC} + \Delta U_{AC} = W_{f,AB} + W_{f,BC}$$

or

$$\frac{1}{2}mv_C^2 = mg(y_A - y_C) - \mu_k mg(y_A - y_B)\cot\theta_{AB} - \mu_k mg(y_B - y_C)\cot\theta_{BC}$$

EVALUATE Solving the equation for v_B, we obtain

$$v_B = \sqrt{2g(y_A - y_B)(1 - \mu_k \cot\theta_{AB})} = \sqrt{2(9.8\text{ m/s}^2)(25\text{ m})[1 - 0.11\cot(32°)]} = 20\text{ m/s}$$

Similarly, for v_C, we have

$$\begin{aligned} v_C &= \sqrt{2g[(y_A - y_C) - \mu_k(y_A - y_B)\cot\theta_{AB} - \mu_k(y_B - y_C)\cot\theta_{BC}]} \\ &= \sqrt{2(9.8\text{ m/s}^2)[63\text{ m} - (0.11)(25\text{ m})\cot(32°) - (0.11)(38\text{ m})\cot(20°)]} \\ &= 30\text{ m/s} \end{aligned}$$

ASSESS Let's consider the case where $\mu_k = 0$. In this limit, the results become

$$v_B = \sqrt{2g(y_A - y_B)} = \sqrt{2(9.8\text{ m/s}^2)(25\text{ m})} = 22\text{ m/s}$$
$$v_C = \sqrt{2g(y_A - y_C)} = \sqrt{2(9.8\text{ m/s}^2)(63\text{ m})} = 35\text{ m/s}$$

which are the same as the result of Problem 5.19 for the frictionless case.

52. **INTERPRET** You're asked to determine the efficiency of a pumped storage facility. If the efficiency were 100%, then all the gravitational potential energy of the water would be converted to electricity. So you have to find what actual percentage of the potential energy is converted to electricity.

DEVELOP Initially, all the water is in the upper reservoir. If we assume the gravitational potential energy is zero at the level of the generating station, then the stored potential energy of the water in the upper reservoir is $U = mgh$. You need to compare this to the electrical energy generated by the station over the given time period: $W = P\Delta t$.

EVALUATE The efficiency is the energy output of the generators divided by the energy input of the water:

$$\varepsilon = \frac{W}{U} = \frac{P\Delta t}{mgh} = \frac{(330\text{ MW})(8.0\text{ h})}{(8.5\times10^9\text{kg})(9.8\text{ m/s}^2)(140\text{ m})} = 81\%$$

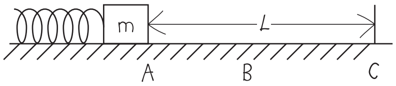
ASSESS The missing energy is lost to non-conservative forces, such as drag forces in pipes that channel the water and friction in the turbines that turn the generators.

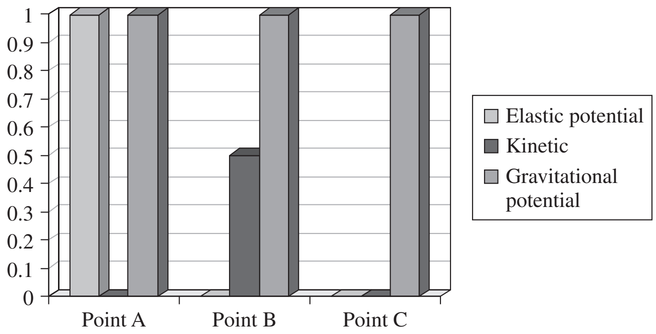
53. **INTERPRET** In this problem we want to find the distance a block slides on surface with friction after being launched by a compressed spring. The force of friction is not conservative, so we will apply the principle that the work done by friction accounts force the change in the mechanical energy (Equation 7.5).

DEVELOP Suppose the block comes to rest at point C (see figure below), which is a distance L from its initial position at rest against the compressed spring at point A. Use Equation 7.5, $\Delta K + \Delta U = W_{\text{nc}}$, where W_{nc} is the work done by nonconservative force (i.e., friction in this problem). This leads to

$$W_{\text{nc}} = -\mu_k mgL = \overbrace{\Delta K}^{=0} + \Delta U = -\frac{1}{2}kx^2$$

because the kinetic energies at A and C and the change in gravitational potential energy are zero.





EVALUATE Solving the above equation, we obtain

$$L = \frac{kx^2}{2\mu_k mg} = \frac{(340\ \text{N/m})(0.18\ \text{m})^2}{2(0.27)(1.5\ \text{kg})(9.8\ \text{m/s}^2)} = 1.4\ \text{m}$$

ASSESS The distance moved by the block is proportional to the spring's potential energy, $kx^2/2$. In addition, it is inversely proportional to the coefficient of friction, μ_k. In the limit $\mu_k = 0$, the surface is frictionless and we expect the mass to travel indefinitely. The exchange between elastic potential, kinetic, and gravitational potential energy can be seen from the bar chart in the figure above. Because the block slides on a horizontal surface, the gravitational potential energy does not change (and we define it arbitrarily to be 1 J). At position A, the block-spring system's energy is entirely elastic potential energy. At point B, the system's energy is kinetic, but friction has already consumed some of the energy, so the kinetic energy is not equal to the initial elastic potential energy. At point C, the block stops, and all its initial elastic potential energy has been consumed.

54. **INTERPRET** This problem involves conservation of energy with conservative and nonconservative forces. The conservative force acting on the bug is gravity, and the nonconservative force is the friction it experiences crossing the sticky patch at the bottom of the bowl. We are to find how many times the bug can slide across the sticky patch before the work done by friction consumes all the bug's initial mechanical energy.

DEVELOP Apply Equation 7.5, $\Delta U + \Delta K = W_{\text{nc}}$. The bug's initial mechanical energy is its gravitational potential energy, $U_0 = mgh$, where $h = 11$ cm. Every time the bug crosses the sticky patch, is loses the energy $W_{\text{nc}} - \vec{f}_{\text{k}} \cdot \vec{d} = -f_{\text{k}} d = -\mu_{\text{k}} mgd$. The ratio of the initial energy to this energy loss is the number of times the bug can cross the sticky patch.

EVALUATE The number of times the bug will cross the sticky patch is

$$n = \frac{U_0}{W_{nc}} = \frac{mgh}{\mu_k mgd} = \frac{(11\ \text{cm})}{(0.61)(1.5\ \text{cm})} = 12.0 \pm 0.5$$

to two significant figures. Therefore, from the given data, we expect that the bug will cross the sticky patch 11 or 12 times.

ASSESS To 3 significant figures, the result is 12.0. Thus, if we knew the data to 3 significant figures, we would be able to give a more precise answer (i.e., 12).

55. **INTERPRET** In this problem we want to find the final position of a block after being launched from a compressed spring. Its path involves a frictional surface followed by a frictionless curve. There forces acting on the block are conservative (gravity and the elastic force) and nonconservative (friction). We will define the block's initial position as the zero of gravitational potential energy.

DEVELOP The energy of the block when it first encounters friction is completely kinetic and, by conservation of total mechanical energy (Equation 7.7) it is equal to the initial elastic potential energy of the block/spring system:

$$K_0 = \frac{1}{2}kx^2$$

Upon crossing the friction zone, the work done by the friction is

$$W_{\text{nc}} = -\mu_k mgL$$

Depending on the ratio of $K_0/|W_{\text{nc}}|$, the block will move back and forth several times before losing all its energy and coming to rest.

EVALUATE Initially the block has an energy

$$K_0 = \frac{1}{2}kx^2 = \frac{1}{2}(200\text{ N/m})(0.15\text{ m})^2 = 2.25\text{ J}$$

The work done by the friction is

$$\Delta E = W_{\text{nc}} = -\mu_k mgL = -(0.27)(0.19\text{ kg})(9.8\text{ m/s}^2)(0.85\text{ m}) = -0.427\text{ J}$$

Because $K_0/|W_{\text{nc}}| = 5.27$, five complete crossings are made, leaving the block with energy $K = K_0 - 5|W_{\text{nc}}| = 0.113\text{J}$ on the curved side. This remaining energy is sufficient to move the block a distance

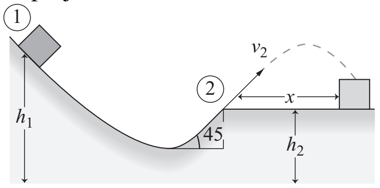
$$s = \frac{K}{\mu_k mg} = \frac{0.113\text{ J}}{(0.27)(0.19\text{ kg})(9.8\text{ m/s}^2)} = 0.225\text{ m}$$

so the block comes to rest 85 cm – 22.5 cm = 62.5 cm to the right of the beginning of the friction patch.

ASSESS Because $K_0 > |W_{\text{nc}}|,$ the block does not lose all its energy the first time when it moves across the frictional zone. No energy is lost while it moves along the frictionless curve. The number of times the block moves back and forth across the frictional zone depends on the ratio $K_0/|W_{\text{nc}}|$.

56. **INTERPRET** All forces involved in this problem are conservative forces, so the problem involves conservation of total mechanical energy. It also involves kinematics (Chapter 3) to find the range of the block.

DEVELOP Use conservation of mechanical energy in the form of Equation 7.6, $\Delta K + \Delta U = 0$. The block starts from rest, so $\Delta K = mv^2/2 - 0 = mv^2/2$. The change in potential energy is $\Delta U = mgh_2 - mgh_1 = mg(h_2 - h_1)$. From Equation 3.15, we find that the range of a projectile launched at 45° is $x = v^2/g$.



EVALUATE Using conservation of total mechanical energy to find the launch speed v2 gives

$$\frac{1}{2}mv_2^2 + mg(h_2 - h_1) = 0$$

$$v_2 = \pm\sqrt{2g(h_1 - h_2)}$$

Inserting this result into Equation 3.15, we find that the range is

$$x = 2(h_1 - h_2)$$

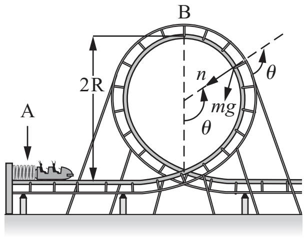
ASSESS The range is simply the difference in height between points 1 and 2. Notice that by conservation of total mechanical energy, we can also say that the maximum height attained by the block above point 2 is $h_1 - h_2$.

57. **INTERPRET** The object of interest is the roller coaster that, after being launched from a compressed spring, moves along a frictionless circular loop. The physical quantity we are asked about is the minimum compression of the spring that allows the car to stay on the track. To find this, we will need to apply Newton's second law as well as conservation of mechanical energy.

DEVELOP If the car stays on the track, the normal force applied by the track must be greater than zero and the radial component of the cars acceleration is $a = v^2/R$. Applying Newton's second law to the roller coaster gives

$$n = \frac{mv^2}{R} + mg\cos\theta \geq 0 \quad \rightarrow \quad v^2 \geq -gR\cos\theta$$

The function $-\cos\theta$ is maximal at the top of the loop (θ= 180°, see figure below), so $v_B^2 \geq gR$ is the condition for the car to stay on the track all the way around. This is the result obtained in Example 5.7.



With the minimum speed at point B determined, apply conservation of total mechanical energy (Equation 7.7), to find the minimum compression length of the spring.

EVALUATE In the absence of friction, conservation of total mechanical energy requires

$$K_A + U_A = K_B + U_B \quad \rightarrow \quad 0 + \frac{1}{2}kx^2 + mgy_A = \frac{1}{2}mv_B^2 + mgy_B$$

Solving for x, we obtain

$$x^2 = \frac{m}{k}\left[v_B^2 + 2g\left(y_B - y_A\right)\right] \geq \frac{5mgR}{k}$$

or

$$x \geq \pm\sqrt{\frac{5mgR}{k}} = \sqrt{\frac{5(840\text{ kg})(9.8\text{ m/s}^2)(6.2\text{ m})}{31{,}000\text{ N/m}}} = 2.9\text{ m}$$

ASSESS Our result indicates that if the radius of the loop increases, then the amount of spring compression must increase in proportion to the square root of the radius for the car to stay on the track. On the other hand, when a stiffer spring (with larger k) is used, then less compression would be required. Notice that we retained the positive sign in our solution because we defined x to be a compression. The negative sign therefore corresponds to an extension of the spring, which gives the spring the same elastic potential energy, but would accelerate the roller coaster in the opposite direction.

58. **INTERPRET** This problem involves the conservative force of gravity, so we can apply conservation of total mechanical energy. We will take the zero of the gravitational potential energy to be the bottom of the bowl. We are asked to find the horizontal coordinate of the points where the particle reverses direction (i.e., the turning points).

DEVELOP To apply conservation of total mechanical energy, we need to express the total mechanical energy at the turning points and at one other position where the total mechanical energy is known. For the turning points, we have $U_0 + K_0 = mgh$ because the particle's kinetic energy is zero at the turning point (because $v = 0$). At the bottom of the bowl, the gravitational potential energy is zero, and all the mechanical energy is in the kinetic form, which is $K = mv^2/2$. Equating the two allows us to find the maximum height the particle attains, from which we can find the horizontal coordinate x of the turning point.

EVALUATE The maximum height attained by the particle is

$$mgh = \frac{1}{2}mv^2 \quad \Rightarrow \quad h = \frac{v^2}{2g}$$

From this result, we find the turning points are

$$x = \pm\sqrt{\frac{h}{0.18\text{ m}^{-1}}} = \pm(0.47\text{ m/s})\sqrt{\frac{1}{2\left(0.18\text{ m}^{-1}\right)\left(9.8\text{ m/s}^2\right)}} = \pm 25\text{ cm}$$

ASSESS The positive and negative signs correspond to the right and left turning points.

59. **INTERPRET** In this problem we want to find the distance a child can move across a frictional surface after sliding down a frictionless incline. The problem involves the conservative force of gravity for the first part (the incline) and the nonconservative force of friction for the second part (the level). Thus, we can apply conservation of total mechanical energy to the incline, and the concept of work done by friction to the level section. We will take the zero of gravitational potential energy to be the bottom of the incline.

DEVELOP At the top of the hill, the child's mechanical energy is entirely gravitational potential energy, so $U_0 + K_0 = mgh$, where h = 7.2 m. At the bottom of the hill, just before starting across the rough surface, all this energy is converted to kinetic energy, so $U + K = K$. By conservation of total mechanical energy, we can equate these two expressions, which gives

$$K = mgh$$

where K is the kinetic (and total) energy at the beginning of the rough section. As the child progresses across the rough surface, this energy is consumed by the work done by friction, and the sled stops when the energy supply is exhausted. This is expressed by Equation 7.5, $\Delta U + \Delta K = W_{nc}$. Because the rough section is level, $\Delta U = 0$, and the work done by friction is $W_{\rm nc} = \vec{f}_{\rm k} \cdot \vec{x} = -\mu_{\rm k} mgx$, so we have

$$\Delta K = K_{\rm final} - K = 0 - mgh = -\mu_{\rm k} mgx$$

EVALUATE Solving the above equation for x, we obtain

$$x = \frac{h}{\mu_k} = \frac{7.2\ \text{m}}{0.51} = 14\ \text{m}$$

ASSESS As expected, the distance the child travels is proportional to h, because the greater is h; the more gravitational potential energy there is to convert to kinetic energy. On the other hand, we expect x to be inversely proportional to the coefficient of friction, $\mu_{\rm k}$. A smaller $\mu_{\rm k}$ will allow the child to travel a much further distance before losing all its kinetic energy. In the limit that $\mu_{\rm k} \to 0$, the child will slide forever, as expected from Newton's second law.

60. **INTERPRET** This problem involves Newton's second law and conservation of total mechanical energy. The force of gravity that acts on the bug is a conservative force, and we shall take the geometric center of the man's spherical head to be the zero of gravitational potential energy.

DEVELOP Draw a diagram of the situation (see figure below). Apply conservation of mechanical energy to express the bug's speed as a function of its vertical position d on the man's head. This gives

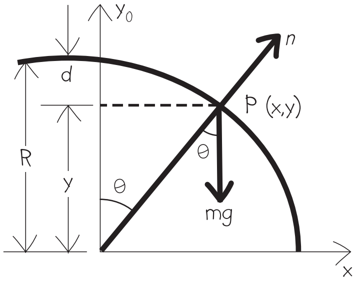
$$U_0 + \overbrace{K_0}^{=0} = U + K$$

$$mgR = mg(R - d) + \frac{1}{2}mv^2$$

The bug will no longer be in contact with the man's head when the normal force goes to zero. From the radial component of Newton's second law, the normal force can be found.

$$n + mg\cos\theta = m\frac{v^2}{R}$$

Combining these equations and letting n = 0, we can solve for d.



EVALUATE Using the fact that $\cos\theta = (R - d)/R$, we have

$$mg\frac{R-d}{R} = m\frac{v^2}{R}$$

$$v^2 = g(R - d)$$

Inserting this result into the result from conservation of energy gives the distance d to be

$$mgR = mg(R-d) + \frac{1}{2}mg(R-d)$$
$$d = \frac{R}{3}$$

ASSESS If the bug is perfectly positioned on the top of the man's head and it doesn't move, it will of course stay there. However, this is an unstable equilibrium, so any slight perturbation will send it sliding down the man's head.

61. **INTERPRET** This problem deals with a conservative, one-dimensional force F(x). We know that this force is conservative because it depends only on position. We can therefore apply the fact that the work done by this force on the particle equates to the loss in the particles potential energy (see Equation 7.2).

DEVELOP Applying Equation 7.2 gives

$$\Delta U = -\int_{x_1}^{x_2} \vec{F}(x)\cdot d\vec{r} = -\int_{x_1}^{x_2} F(x)\,dx$$

Because the force F(x) is conservative, we can apply conservation of total mechanical energy (Equation 7.7), $U_0 + K_0 = U + K$. The subscript 0 refers to the initial state, so $K_0 = 0$ because the particle is initially at rest. Once $\Delta U = U - U_0$ is known, the speed of the particle can be calculated from

$$K = \frac{1}{2}mv^2 = -\Delta U$$

EVALUATE Integrating $F(x) = a\sqrt{x}$, we obtain

$$\Delta U = U - U_0 = -\int_0^x a\sqrt{x'}\ dx' = \frac{2a}{3}(x')^{3/2}\bigg|_0^x = \frac{2a}{3}x^{3/2}$$

Therefore, the speed of the particle as a function of x is

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{-2\Delta U}{m}} = \left(\frac{4a}{3m}x^{3/2}\right)^{1/2} = 2x^{3/4}\sqrt{\frac{a}{3m}}$$

ASSESS We can check our answer by substituting the result back to the expression for K. This leads to

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{4a}{3m}x^{3/2}\right) = \frac{2a}{3}x^{3/2}$$

Indeed, we see that $K = -\Delta U$, as required by the principle of conservation of energy.

62. **INTERPRET** This problem involves only conservative forces (i.e., gravity), so we can apply conservation of total mechanical energy. We will take the zero of the gravitational potential energy to be the ground level. We are asked to find the speed at which Tarzan must run so that he swings just far enough to cross the gorge, so that he can just let go and drop vertically down on the far side.

DEVELOP Tarzan's initial mechanical energy is completely kinetic, so $U_0 + K_0 = mv_0^2/2$. His final mechanical energy will be completely due to gravitational potential energy, so $U + K = mgy$. Using trigonometry, the final height h can be expressed in terms of the width w = 10 m of the gorge and the length l = 17 m of the vine: $h = l(1 - \cos\theta)$, where $\text{asin}(d/l) = \theta$.

EVALUATE Equating the initial and final energies, and solving for the speed v gives

$$\frac{1}{2}mv_0^2 = mgy = mgl(1-\cos\theta) = mgl\left(1 - \frac{\sqrt{l^2-d^2}}{l}\right)$$

$$v_0 = \pm\sqrt{2gl\left(1-\frac{\sqrt{l^2-d^2}}{l}\right)} = \pm\sqrt{2(9.8\ \text{m/s}^2)(17\ \text{m})\left(1-\frac{\sqrt{(17\ \text{m})^2-(10\ \text{m})^2}}{17\ \text{m}}\right)} = 8.0\ \text{m/s}$$

to two significant figures and where we have chosen the positive sign to indicate that Tarzan must run to the right.

ASSESS If we let the gorge shrink to $d = 0$, we find that v_0 goes to zero, as expected. The two signs indicate that the speed can be to the right or to the left, but we have chosen the positive direction to be to the right.

63. **INTERPRET** We find whether a spring-launched block makes it to the top of an incline with friction, and how much kinetic energy it has when it gets there (if it gets there.) We can use energy methods to solve this problem, but friction is a factor so mechanical energy is not conserved.

DEVELOP The initial energy of the system is spring potential energy: $U_\text{i} = \frac{1}{2}kx^2$. This energy is converted to kinetic energy and gravitational potential energy, but some energy is lost to (non-conservative) friction. From Equation 7.5, we can write this as:

$$U_\text{i} = K + U_g + W_\text{nc}$$

The gravitational potential energy is related to the height, h, the block reaches: $U_g = F_g h$. However, the work done by friction is related to the distance, Δs, the block travels along the incline: $W_\text{nc} = f\Delta s$, where the friction force is given by $f = \mu F_g \cos\theta$. The two distances are related by: $h = \Delta s \sin\theta$.

EVALUATE We can determine whether the block makes it to the top of the incline by setting $\Delta s = L$, the length of the incline, and solving $K = U_\text{i} - U_g - W_\text{nc}$. If $K \geq 0$, then the block does reach the top. Otherwise it doesn't, and we can solve for the distance where its kinetic energy drops to zero. So solving for the kinetic energy at the top:

$$\begin{aligned} K &= \tfrac{1}{2}kx^2 - F_g L(\sin\theta + \mu\cos\theta) \\ &= \tfrac{1}{2}(2.0\text{ kN/m})(10\text{ cm})^2 - (4.5\text{ N})(2.0\text{ m})\left(\sin 30^\circ + 0.50\cos 30^\circ\right) = 1.6\text{ J} \end{aligned}$$

So yes, the block reaches the top with 1.6 J of kinetic energy.

ASSESS Another way to solve this problem is to set $K = 0$ and then solve for the distance that the block would have to travel, Δs, before coming to rest. If $\Delta s \geq L$, then the block reaches the top of the incline with kinetic energy to spare.

64. **INTERPRET** How long will it take to accelerate from zero to 60 mph? We are given the power of the car, the efficiency, and the mass. When the car travels on a level road, the work done equals the change in kinetic energy (Equation 7.5). Power is work per time, so we can find the time from the power and the work.

DEVELOP Convert the given power of the car (250 hp) to Watts, using $1\text{ hp} = 746\text{ W}$, recalling that the power available is only 30% of the engine horsepower. Also convert the final speed from mph to m/s, using $1\text{ mph} = 0.447\text{ m/s}$. Use $P = W/t = \Delta K/t$, and solve for the time t.

EVALUATE The power of the car is $P = (250\text{ hp})\times(30\%)\times(746\text{ W/1 hp}) = 56\text{ kW}$. The final speed of the car is $v = (60\text{ mph})\times(0.447\text{ m/s/1 mph}) = 26.8\text{ m/s}$. The time to reach this speed is

$$P = \frac{W}{t} = \frac{\Delta K}{t}$$

$$t = \frac{\Delta K}{P} = \frac{\frac{1}{2}mv^2}{P} = \frac{(1500\text{ kg})(26.8\text{ m/s}))^2}{2\left(56\times10^3\text{ W}\right)} = 9.6\text{ s}$$

ASSESS This is not particularly high performance, but it's a reasonable 0-to-60 time for an automobile.

65. **INTERPRET** This problem involves the conservative forces of a spring and gravity (of the Moon, in this case). We can therefore apply conservation of total mechanical energy to find the requisite spring constant. We will take the surface of the Moon to the zero of gravitational potential energy.

DEVELOP To apply conservation of total mechanical energy, we need to express the initial energy, when the spring is compressed, and the final energy, just when a bin is separating from the spring. The initial energy is $U_0 + K_0 = ky^2/2$, and the final energy is $U + K = mg_\text{M}y + mv^2/2$. Equating the two allows us to solve for the spring constant k.

EVALUATE

$$\frac{1}{2}ky^2 = mg_\text{M}y + \frac{1}{2}mv^2$$

$$k = \frac{2mg_\text{M}}{y} + \frac{mv^2}{y^2} = \frac{2(1000\text{ kg})\left(1.6\text{ m/s}^2\right)}{15\text{ m}} + \frac{(1000\text{ kg})(2400\text{ m/s})^2}{(15\text{ m})^2} = 2.1\times10^2\text{ N/m} + 2.6\times10^7\text{ N/m} = 2.6\times10^7\text{ N/m}$$

where we have used the acceleration of gravity on the Moon's surface from Appendix E.

ASSESS This is an extraordinarily strong spring—some two orders of magnitude larger than the effective spring constant in a carbon monoxide molecule (cf. Problem 32). Note also that the contribution of the Moon's gravity in our calculation is negligible, being five orders of magnitude less than the contribution of the spring.

66. **INTERPRET** This problem involves conservative forces (i.e., gravity), so we can apply conservation of total mechanical energy. We are to derive a "leaping equation" that relates the power of the animal to its mass, the push-off distance, and the height reached. We will take the ground to be the zero of gravitational potential energy.

DEVELOP The height when the animal leaves the ground is d, and the final height attained in the jump is h. To apply the conservation of total mechanical energy, we equate the mechanical energy at the heights d and h. At d, the energy is $U_0 + K_0 = mgd + mv^2/2$, and at h the energy is $U + K = mgh + 0$. From Equation 6.12 we know that the change in kinetic energy equates to the *net* work done, which is the work done by the animal plus the work done by gravity. Thus, we know that to accelerate from zero to v, the animal must do work given by

$$W_{\text{net}} = W_{\text{anmial}} + \overbrace{W_{\text{gravity}}}^{=-mgd} = \Delta K = \frac{1}{2}mv^2 - 0$$

$$W_{\text{animal}} = \frac{1}{2}mv^2 + mgd$$

where the work done by gravity is negative because the force of gravity acts to oppose the animal's upward movement. Power is work per unit time ($P = W/t$), and we use Equation 2.9:, $d = (v_0 + v)t/2$ to find t, with $v_0 = 0$, so $t = 2d/v$.

EVALUATE Conservation of mechanical energy gives the kinetic energy at d:

$$mgd + \frac{1}{2}mv^2 = mgh \quad \Rightarrow \quad v = \pm\sqrt{2g(h-d)}$$

Power is

$$P = \frac{W_{\text{anmial}}}{t} = \frac{mv^2/2 + mgd}{2d/v} = \frac{mgh}{2d}\sqrt{2g(h-d)}$$

ASSESS We see that the power depends linearly on the mass, and in a more complex manner on the h and d. We'd better check units on this equation:

$$P = \frac{\text{kg}\cdot\text{m}\cdot\text{s}^{-2}\cdot\text{m}}{\cancel{\text{m}}}\sqrt{\cancel{\text{m}}\cdot\text{s}^{-2}\cdot\cancel{\text{m}}} = \overbrace{\underbrace{\left(\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\right)}_{}^{=\text{N}}\cdot\text{m}}^{=\text{J}}\cdot\text{s}^{-1} = \text{J/s}$$

It's good!

67. **INTERPRET** The energy stored in the artificial tendon is given by Equation 7.2a for a one-dimensional force. What's unique in this problem is that the force suddenly changes when the second spring is engaged.

DEVELOP When only one spring is engaged, the force exerted by the artificial tendon is $F = -kx$ for $x = 0$ to $x = x_1$. But then for $x > x_1$, the second spring engages, thus increasing the force to $F = -(k + ak)x$. Therefore, in doing the integral of Equation 7.2a, we should divide it into two parts:

$$\Delta U = -\int_0^{x_1} F(x)\,dx - \int_{x_1}^{x_2} F(x)\,dx$$

EVALUATE Performing the integration, we find

$$\Delta U = \tfrac{1}{2}kx^2\Big|_0^{x_1} + \tfrac{1}{2}k(1+a)x^2\Big|_{x_1}^{x_2} = \tfrac{1}{2}kx_2^2 + \tfrac{1}{2}ka\left(x_2^2 - x_1^2\right)$$

ASSESS The first term in our result is the energy stored in the first spring, and the second term is the energy stored in the second spring.

68. **INTERPRET** We are asked to analyze a graph characterizing the potential energy between two deuterons.
DEVELOP From Equation 7.8: $F_x = -dU/dx$, the force is zero where the slope of the $U(x)$ curve is zero.

EVALUATE There are two points where the potential appears to have zero slope: a minimum at around 1 fm and a maximum at 5 fm. Only this second point is one of the choices.
The answer is (c).

ASSESS The peak at 5 fm is an unstable equilibrium. If we were to calculate the force at a point near the peak, we would find that the force's direction is away from the peak. It's similar to placing a ball at the top of a hill; any slight movement will cause it to roll down the hill. Contrast this with the potential "well" at 1 fm. Here, any deviation from the well's bottom will cause a force that pulls the deuterons back toward the minimum energy position.

69. **INTERPRET** We are asked to analyze a graph characterizing the potential energy between two deuterons.
DEVELOP When the deuterons are far apart, their potential energy is zero, $U_0 = 0$, but they will have some initial kinetic energy, K_0. We assume the deuterons are moving towards each other. As the distance between the deuterons shrinks, the graph shows that the potential energy increases. According to the conservation of mechanical energy (Equation 7.7), the kinetic energy will correspondingly decrease: $K(x) = K_0 - U(x)$. The question, then, is will the kinetic energy go to zero before the deuterons reach the well in the potential at $x \approx 1$ fm, where they will be fused (bound) together?

EVALUATE The deuterons won't be able to fuse if they run out of kinetic energy before reaching the peak at 5 fm in the potential energy curve. In other words, the initial kinetic energy has to be greater or equal to this "energy barrier" (i.e. $K_0 \geq U_{\text{peak}}$). The potential energy at the peak is equal to about 0.3 MeV.
The answer is (d).

ASSESS We recall once again the analogy to a ball and a hill. If the ball starts at the bottom of the hill, it will only be able to reach the peak if its initial kinetic energy is greater or equal to the potential energy separating the top and bottom of the hill.

70. **INTERPRET** We are asked to analyze a graph characterizing the potential energy between two deuterons.
DEVELOP As was said in the previous problem, the potential energy is zero when the deuterons are widely separated. If they fall into the potential well at 1 fm, they will be bound together into a single nucleus (in this case a helium nucleus).

EVALUATE The potential energy at the bottom of the potential well is around -3.3 MeV. Therefore, the energy available in fusion is $0-(-3.3 \text{ MeV}) = 3.3 \text{ MeV}$.
The answer is (c).

ASSESS The available energy from deuteron fusion is in the form of kinetic energy. I.e., the resulting helium nucleus will have kinetic energy of 3.3 MeV. Suppose that many of these fusion reactions are occurring inside a star or a reactor chamber, then all the kinetic energy that is released can be considered as heat.

71. **INTERPRET** We are asked to analyze a graph characterizing the potential energy between two deuterons.
DEVELOP The force is given by Equation 7.8: $F_x = -dU/dx$.

EVALUATE The slope of the curve at 4 fm is positive, so the force is negative. That means the force points toward smaller x, which means it is pulling the deuterons closer together. This is an attractive force.
The answer is (b).

ASSESS We shouldn't confuse the magnitude of the force with the fact that the potential, $U(x)$ is zero at $x = 4$ fm. An attractive force is consistent with the notion that the deuterons are bound to each other inside the potential well. The force at $x = 4$ fm is pulling the deuterons back to the equilibrium position at $x \approx 1$ fm, like a stretched spring.